

ISEG - Quantitative Finance Formulae

Quantitative Finance Formulas

Interest accumulation: $FV = PV + I$

Simple interest: $FV = PV(1 + i \cdot t)$

Compound interest: $FV = PV(1 + i)^t$

Simple discount: $D = FV \cdot d \cdot t$

I =Interest; P =Principal; i =interest rate

t =number of periods

Effective rates conversion:

$$i_L = (1 + i_S)^{1/S} - 1; i_S = (1 + i_L)^{S/L} - 1$$

Relation between nominal and effective rates:

$$i_A(m) = m[(1 + i_A)^{1/m} - 1]$$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S = Pe^{\delta t}$

Present Value: $P = Se^{-\delta t}$

Present value of a n payment annuity immediate of

$$1 \text{ per period: } a_{\bar{n}|i} = \frac{1 - (1+i)^{-n}}{i}$$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\bar{n}|i} = \frac{(1+i)^n - 1}{i} = a_{\bar{n}|i}(1+i)^n$$

Present value of annuity due:

$$\ddot{a}_{\bar{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\bar{n}|i}(1+i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\bar{n}|i} = s_{\bar{n}|i}(1+i)$$

Present value of deferred annuity:

$${}_k|a_{\bar{n}|i} = a_{\bar{n}|i}(1+i)^{-k}$$

Accumulated value of deferred annuity:

$${}_k|s_{\bar{n}|i} = s_{\bar{n}|i}$$

Forborne annuities

$$FV = R \cdot S_{n|i}(1+i)^p$$

p - number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\infty|i} = \frac{1}{i}$

Increasing arithmetic progression:

$$(C - h)a_{\bar{n}|i} + h(1a)_{\bar{n}|i}; \quad (1a)_{\bar{n}|i} = \frac{a_{\bar{n}|i} - n(1+i)^{-n}}{i}$$

Decreasing arithmetic progression:

$$(D - h)a_{\bar{n}|i} + h(Da)_{\bar{n}|i}; \quad (Da)_{\bar{n}|i} = \frac{n - a_{\bar{n}|i}}{i}$$

Geometric progression: $C \frac{1 - r^n(1+i)^{-n}}{1+i-r}$

M^{thly} payable annuity:

$$a_{\bar{n}|i}^{(m)} = a_{\bar{n}|i(m)} \frac{i}{i(m)}; \quad s_{\bar{n}|i}^{(m)} = s_{\bar{n}|i(m)} \frac{i}{i(m)}$$

Leasing:

Lease payment = PMT + I

$$PV = PMT a_{\bar{n}|i}, \quad I = RV \cdot i$$

Leasing (for an annuity immediate):

$$VC = E + Ra_{\bar{n}|i} + RV(1+i)^{-n}, \text{ where}$$

VC : value of the contract; E : entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

$$R_n = R_1 + [(R_2 - R_1) / (t_2 - t_1)] \cdot (t_n - t_1)$$

R_n - unknown rate

R_1 and R_2 - two known